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Wang Xikang

Central Research Institute of Building & Construction of MMI, China

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Wang Xikang
Senior Engineer, Central Research Inst. of Building & Construction, Ministry of Metallurgical Ind., Beijing, China

SYNOPSIS: On the basis of many tests it has been proved that the damping properties of bases not only Have bearing on magnitude of base area of foundation but also on weight of foundation and on mechanics model adopted. This paper puts forward the computation formulas of damping parameters to some soils.

INTRODUCTION

In order to compute the vibration response of structures and foundations under machinery with satisfactory precision it is necessary to establish the damping properties of bases correctly. In recent years the problem above mentioned has been researched by the author and his colleagues under the conditions of field tests employing vertical forced vibration with smaller amplitude which were performed on foundation of different sizes and different soil conditions. When researching the damping properties of bases a new concept is cited, Whereby the mass participating in foundation vibration includes not only mass of foundation but also soil mass. In consequence of considering the soil mass not only the damping parameters established agree with practice to a greater extent but also the precision of computation of vibration reaponse acquires great improvement. In the following several main results of research are introduced in detail.

1. MECHANICS MODEL AND THEORETICAL FORMULAS FOR DAMPING COMPUTATION

Before researching the damping properties of bases the mechanics model of foundation vibration must be first developed because using the different mechanics model will obtain different dynamic parameters. When analysing the damping properties of bases, the model of Mass-Damping-Spring of linearity is adopted as shown in Fig.1 and the following assumptions are employed:

1) Base soils possess not only elasticity but also inertia effect.

2) The soil mass participating in vibration and vertical damping coefficient as well as vertical dynamic rigidity of bases are not varied within the range of exciting frequencies.

Because three dynamic parameters which are dynamic rigidity, damping coefficient and soil mass must be determined the model may be referred to as three parameter model. The reasonableness of adopting three parameter model had been discussed in detail in the paper 2 and is not discussed herein.

Under the harmonic vertical exciting force with constant amplitude as shown in Fig.1 the equation of motion of foundation may be written as follows:

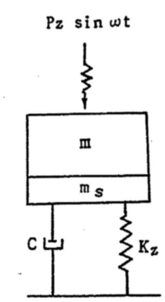

Fig.1 Vibration Model Considering Soil Mass

$$(m+m_s)\ddot{Z}+C\dot{Z}+K_zZ = P_z\mathrm{Sin}\,\omega t \qquad (1)$$

where: m and m_s = Mass of foundation and soil

P_z and ω = Maximum value and frequency of exciting force

C = Vertical damping coefficient

K_z = Dynamic rigidity of base

Z = Vertical displacement of foundation

Limiting our discussion to forced vibration merely, we take the solution of the question in the form:

$$Z = A_z\mathrm{Sin}(\omega t-\theta) \qquad (2)$$

where: A_z = Amplitude of vibration

θ = Phase shift

In order to obtain three dynamic parameters we may substitude the equation 2 into equation 1, and use three known conditions which are resonance frequency, resonance amplitude and amplitude of vibration or phase shift corresponding to a certain exciting frequency. In this paper the first three conditions are employed. When using the three conditions, we may obtain the computing formula for damping ratio D_z of bases as follows:

$$D_z = 0.707\sqrt{1-\sqrt{1-d}} \qquad (3)$$

where:

$$d = \frac{(1-\alpha)^2}{\alpha^2+\beta^2-2\alpha} \qquad (4)$$

$$\alpha = \frac{\omega^2}{\bar{\lambda}_{zs}^2} \qquad (5)$$

$$\beta = \frac{A_{z,max}}{A_z} \qquad (6)$$

$A_{z,max}$ = Resonance amplitude

A_z = Amplitude corresponding to a certain frequency

$\bar{\lambda}_{zs}$ = Resonance frequency

If the maximum value of exciting force is varied with exciting frequency, that is $P_z = m_o r\omega^2 \mathrm{Sin}\,\omega t$ (where m_o is eccentric mass, r is eccentricity), then the damping ratio of base is determined as follows:

$$D_z = 0.707\sqrt{1-\sqrt{1-\tilde{d}}} \qquad (7)$$

where:

$$\tilde{d} = \frac{(1-\alpha)^2}{\alpha^2\beta^2-2\alpha+1} \qquad (8)$$

It should be pointed out that the dynamic rigidity of base and soil mass participating in vibration can be immediately determined after obtaining the damping ratio[2].

In consideration of requirement of following discussion, the computation formula of damping ratio without considering the soil mass is written as follows [3]:

$$D_z = \frac{1-\sqrt{1-\xi_z}}{2} \qquad (9)$$

where:

$$\xi_z = \frac{1}{1+\beta z^2} \qquad (10)$$

$$\beta_z = \frac{A_{z,max}}{P_z} m\bar{\lambda}_{zs}^2 \qquad (11)$$

It should be pointed out that the formula 9 is derived according to following two conditions which are resonance amplitude and resonance frequency acquired. According to the above computation formulas of damping ratio the author analysed over 40 test data and obtained a several important phenomena as follows.

2. THE LAW OF DAMPING of VIBRATION OF BASE

1) The obtained damping ratio according to three parameter model is significantly smaller than that according to two parameter model. To explain this point 8 analysed results are cited among many results. They are listed in Table I.

TABLE I, the Effect of Soil Mass Considered on Damping Ratio

Number of Foundations	Ignoring Soil Mass		Considering Soil Mass	
	m (t)	D_z	$m + m_f$ (t)	D_z
1	20.1	0.162	26.3	0.122
2	16.6	0.291	45.4	0.119
3	41.4	0.243	72.6	0.417
4	44.9	0.217	69.4	0.145
5	22.1	0.125	35.5	0.079
6	8.3	0.123	16.1	0.065
7	24.8	0.172	31.2	0.137
8	9.4	0.169	22.0	0.074

It is clearly seen from the Table that the damping ratio taking into account the soil mass is 30 - 60% smaller than that without considering the soil mass. It is because the dynamic rigidity of bases having considered the soil mass has been markedly increased. Dynamic rigidity and soil mass participating in vibration are discussed in detail[2]. It Should be pointed out that when computing these dynamic parameters it is necessary to use the frequencies beyond the range from 0.8 to 1.2 . Because within this range the accuracy of amplitude of vibration obtained corresponding to a certain frequency is over-sensitive to dynamic parameters.

2) The vertical damping coefficient of bases is apparantly related to the weight of foundation and the weight acting on the foundation. The important phenomena had been neglected in the past, e.g. the literature [4] showed that the damping coefficient of bases is related to rigidity of bases only and has nothing to do with the weight of foundation. Furthermore, the literature 4 gave the damping constants under different bases. Several American experts have not paid sufficient attention to the effect of foundation weight on damping coefficient as well. For simplification the experts F.E.Richart, Jr., and R.V. Whitman put forward the following equation of motion 12 to replace elastic semi-infinite body theory[5]:

$$m\ddot{z} + \frac{3.4}{1-\mu} r_o^2\sqrt{G\rho}\,\dot{z} + \frac{4Gr_o}{1-\mu} Z = P_z \mathrm{Sin}\,\omega t \qquad (12)$$

where: m = Mass of foundation

μ = Poisson's ratio

ρ = Density of soil

G = Elastic shear modulus of soil

r_o = Equivalent radius, $r_o = (F/\pi)^{0.5}$

F = Base area of foundation

From expression 12 we obtain the damping coefficient C and damping ratio D_z as follows:

$$C = \frac{3.4}{1-\mu} r_o^2\sqrt{G\rho} \qquad (13)$$

$$D_z = \frac{C}{C_c} = \frac{0.85}{(1-\mu)^{0.5}} \frac{1}{\sqrt{b_o}} \qquad (14)$$

where $b_o = m/\rho r_o^3$ and ρ is the density of soil.

In this paper parameter b is used. The b is equal to $0.18b_o$.

It is seen from expressions 13 and 14 that the damping parameters have no bearing on weight of foundation. But lots of tests demonstrate that the damping of base is apparantly related to the weight of foundation acting on the soil. The greater weight the greater damping of base. For instance, under the same base area of $16m^2$ the damping coefficients increase from 2767 to 5136 kN.S/m corresponding to increase of foundation weight from 520 to 1000 kN. It may be seen from Table II.

TABLE II. Relationship between Damping Coefficient and Foundation Weight

Foundation Weight (kN)	Base Area of Foundation (m^2)	Damping Ratio (kN.s/m)
130	4.0	1213
226		1772
322		1700
418		1777
292	9.0	2038
485		2729
677		3206

Of course, it is no doubt that the damping coefficients depend on the magnitude of base area of foundation, which agrees to basically theoretical results. It is shown in Table III. clearly.

3) The damping coefficients determined by three parameter model are much smaller than those by the expression 12 which corresponds to the elastic semi-infinite body theory under certain assumption as shown in Table II. Because the damping values determined by semi-infinite body theory are too large, the computing results of amplitude of vibration often do not coincide with the fact. This conclusion has been well proved bu many researchers.

TABLE III. the Coefficient C Established by Two Mechanics Model

Soil	Equivalent Radius r_o (m)	Damping coefficient of Base: from three parameter Method	Damping coefficient of Base: from Expression13
Grave-sand	0.902	797	1610
	1.467	2006	4264
Medium sand	0.902	562	961
	1.130	1213	1703
	1.690	2038	3843
	2.260	5136	6832

4) The damping ratio depends on the embedded depth h_m of foundation. The larger the embedded depth the greater the damping ratio. This conclusion is drawn not only by the author but also by other experts. But the relationship of magnitude of D_z to embedded depth h_m has not been established yet. The relationship of magitude between D_z and h_m is shown in Fig.2 and expression 17. To clarify the effect of embedded depth of foundation on damping ratio 16 test foundations were built on clay loam and slight clay loam by the author and his colleagues.

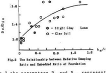

Fig.2 The Relationship between Relative Damping Ratio and Embedded Ratio of Foundation

In Fig.2 the parameters D_z and D_{zo} represent damping ratio corresponding to embedded depths h_m and zero respectively.

5) The tests reveal that the variation law of damping ratio D_z eastablished by expression 12 with parameter b is markedly different from test results. Deviation of theory from practice is described in Fig.3. It should be noted that all these test results shown in Fig.3 were obtained under the condition of loessial clay loam with $\mu = 0.33$. In the figure the parameter b is equal to $w/\gamma F^{1.5}$, where W is weight of foundation, γ unit weight of base soil and F base area of foundation. Fig.3 shows that the damping ratio determined by expression 14 is not only too large but also its variation with b is oversharp. From this we should understand that it is not suitable to use the mechanics model of homogeneous isotropic elastic semi-infinite body instead of practical nonhomogeneous isotropic body.

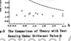

Fig.3 The Comparison of Theory with Test Results Under Different Value b

3 EMPIRICAL FORMULAS OF COMPUTING DAMPING RATIO

All tests presented above demonstrate that the variation of damping ratio is very complicated because the parameter is strongly influenced by many factors such as the influences of stratification of base, level of ground water and so on. But on the basis of these tests it is possible to put forward an appoximate computing formula under the condition of small strain according to three parameter model.

1) Exposed Foundation

The author altogether analysed 38 test data under various bases with Poisson's ratio ranging from 0.32 to 0.4 and shear modulus of elasticity from 39.2 Mpa to 176.4 Mpa. All of the test results are described in Fig.4.

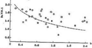

Fig.4 The Relationship between Damping Ratio and Value b under Different Soils

○ Clay Soil ⊙ Medium Sand
⊘ Gravel Sand ⌑ Loessial Clay Loam
⊗ Clay Loam ⌾ Residual Soil

According to these test data the damping ratio may be computed by the following formula:

$$D_{zo} = 0.16/\sqrt{0.3 + b} \qquad (15)$$

The formula 15 is basically suitable for these soils described in Fig.4. As for slight clay loam the damping is markedly smaller as compared with other soils as shown in Fig.5. The damping ratio may be approximately determined by expression 16

$$D_{zo} = \frac{0.11}{\sqrt{0.3 + b}} \qquad (16)$$

It should be pointed out that the dashed line in Fig.4 represent computing results according to formula 15.

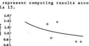

Fig.5 The Relationship between Slight Clay Loam and Value b

2) Embedded Foundations

Almost all of machine foundations and structures are embedded, hence the study of variation law of damping under this kind of condition has more practical meaning. The relative damping ratio for the soils described in Fig.2 may be written as follows:

$$\frac{D_z}{D_{z0}} = 1 + 0.7\sqrt{hm/r_o} \qquad (17)$$

The results of computation of the formula has been represented by solid line in Fig.2.

CONCLUSIONS

Through the analysis above a several major conclusions can be drawn as follows:

1) The magnitude of damping coefficient is related to mechanics model adopted. The value using three parameter model is smaller than that using two parameter model or semi-infinite body model.

2) The damping coefficient of bases is markedly related to the weight of foundation acting on base.

3) The magnitude and variation law of damping established by semi-infinite body theory deviates markedly from the test results.

4) In order to improve the properties of damping of vibration system it is an ideal way to use hollow deep foundations.

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